

## ANALYSIS ON PLANAR ENTRY CONVERGING FLOW OF POLYMER MELTS

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### Abstract

The simulation on the planar entrance convergence flow of polymeric melts was made by means of finite element method (FEM) and the ANSYS software in the present paper. The velocity distributing, the variation of extension rate, shear rate, and the half natural convergence angle ( $\alpha_0$ ) of the fluid were discussed under different initial velocities ( $V_0$ ). The results showed that the velocity of axes flow increased non-linearly with increasing the axial flowing distance and reached the maximum at the entrance. The shear rate increased non-linearly with increasing the radius distance from the center axes of the flow channel, and it reached the maximum at the entrance. The  $\alpha_0$  reduced while the shear rate increased non-linearly with an increase of  $V_0$ , and the  $2\alpha_0$  value of the three-dimension simulation was lower than two-dimension simulation under the same  $V_0$ . Moreover, the simulations were close to the experimental observations and theoretical estimations reported in the literature.

### 1. Introduction

When a polymer fluid (including melt and liquid) flows into a small channel from a reservoir, an entrance convergence flow will be generated because of sudden contraction of the channel and the viscoelastic

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properties of the fluid [11]. That is, the streamline will be not parallel and a conic boundary will be formed due to the viscoelasticity of the fluid and the contraction of the channel. The convergence flow is one of the common flow patterns in the processing and forming of polymer materials, such as moulding, extrusion or injection. It is, therefore, very useful and meaningful in both theory and engineering for understanding the origin and mechanisms of the entrance converging flow, and for promoting the development of rheology theory and instructing polymer processing technology as well as mold design that the entrance convergence flow and flow patterns of polymeric materials, as well as the affecting factors of them are studied deeply [14].

Since 1970's, the converging flow of non-Newtonian fluids have been extensionally studied from different approaches, and some meaningful findings in both theory and practice have gained [2, 6, 9, 18, 20, 22]. Recently, the effects of the elongation viscosity in contraction flow of polymeric melts [4, 16], the relationship between the converging flow and critical conditions for unstable flow [17], and the rheometer for equibiaxial and planar elongations of polymer melts [5] were investigated. In 1990, the author [8] derived a mathematical model of entry converging flow boundary streamline of the non-Newtonian fluids by means of the minimum energy theory in calculus of variations. Binding [1] studied the contraction flow and converging flow of non-Newtonian fluids by means of an approximate analysis method, and proposed an expression on the vortical region length in the front zone of the inlet. In the previous work, the author [10] studied the converging flow of non-Newtonian fluids through an abrupt contraction, and investigated the entry flow patterns at various extrusion rates.

Experimental observation is an important way for studying the entry flow patterns of polymer fluids [3, 12, 19]. In the previous work, the author [12] observed the entry converging flow patterns in die extrusion flow of low-density polyethylene (LDPE) melt using a flow visualization technique, and found that, there was an obvious circulation phenomenon at the front zone of the entrance. Ma and his colleagues [15] investigated

the influence of flow rate on the flow patterns of elastomers and their carbon black compounds during extrusion through dies by using tracer technique. The results showed that the entrance converging angles of the compounds decreased with increasing flow rates. Liang [13] estimated the entry natural converging angles during capillary extrusion flow of carbon black filled natural rubber (NR)/styrene-butadiene rubber (SBR) compound based on the experimental data.

In general, the entry convergent flow is a kind of complicated flow of polymeric materials owing to their viscoelasticity, hence the mathematical model might be very complex and it is difficult to resolve accurately. It is, therefore, numerical simulation has become a usually method to describe roughly the entry converging flow of non-Newtonian fluids recent twenty years [7]. The objective in this paper is to analyze the planar entrance convergence flow during circle die extrusion of polymeric melts by means of a finite element method.

## **2. Theory Analysis**

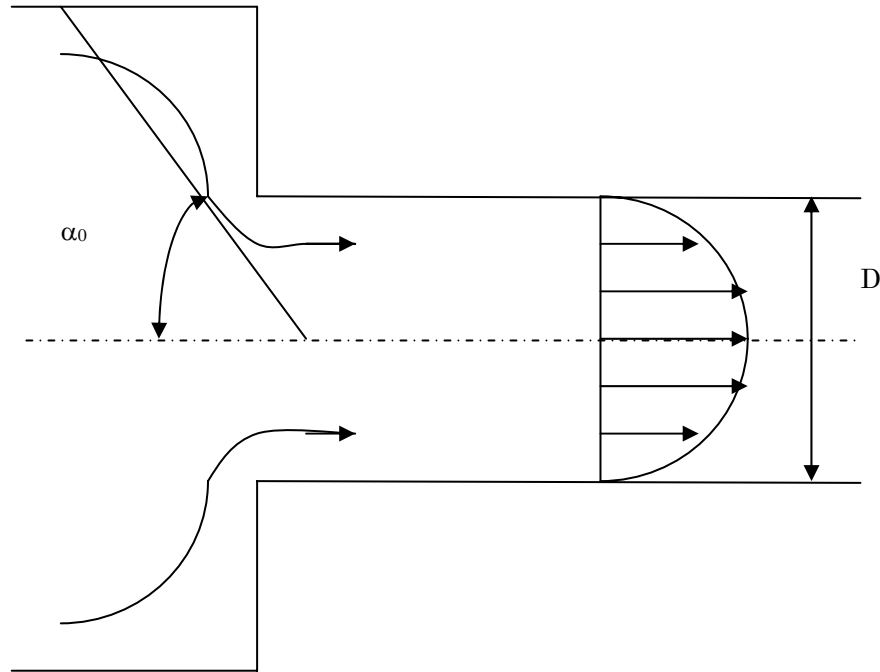
### **2.1. Assumptions**

To investigate expediently the influence of initial velocity of the entry front district of the die on the entry converging flow patterns during extrusion of polymeric melts when temperature is 90°C, the theoretical analysis in this article is based on the following assumption conditions: (1) the fluid flow is isothermal in channel, and the transfer or loss of heat can not be considered; (2) the fluid is incompressible; (3) there is no wall slip phenomenon in the flow; (4) the influence of inertia and gravity force on the flow can be neglected.

### **2.2. Theory description**

Figure 1 shows the sketch of the planar entrance convergence flow of a viscoelastic fluid. It can be seen that the convergence flow is formed at the die entrance when the fluid enters a small channel from a reservoir, and the convergence boundary streamline is generated, which it looks

like as cup shape. The angle between the tangent of the convergence boundary streamline and the center axes of the channel is defined as the half natural converging angle ( $\alpha_0$ ). As stated above, the entry converging flow of the viscoelastic fluid is quite complicated, it includes the extensional flow and shear flow from the viewpoint of dynamics, and the non-constant flow under Lagrange meaning. As to a viscoelastic fluid flows in short die, entrance convergence flow will affect directly the flow state in the exit region.

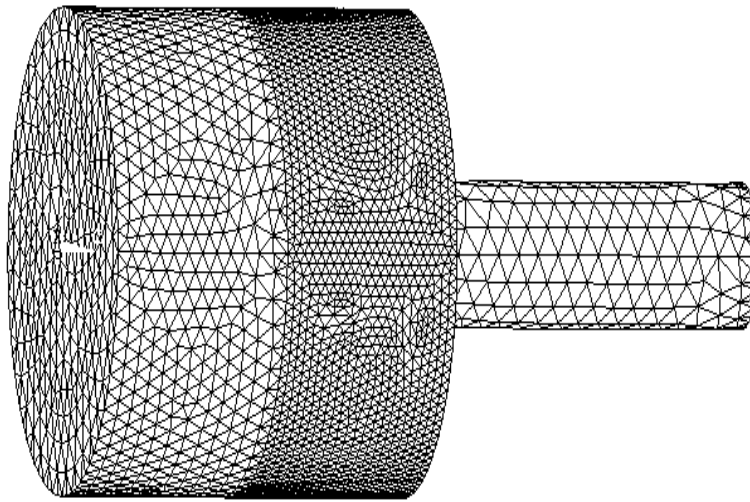


**Figure 1.** Sketch of planar entry convergent flow of polymer melt.

### 3. Modelling

#### 3.1. Meshing

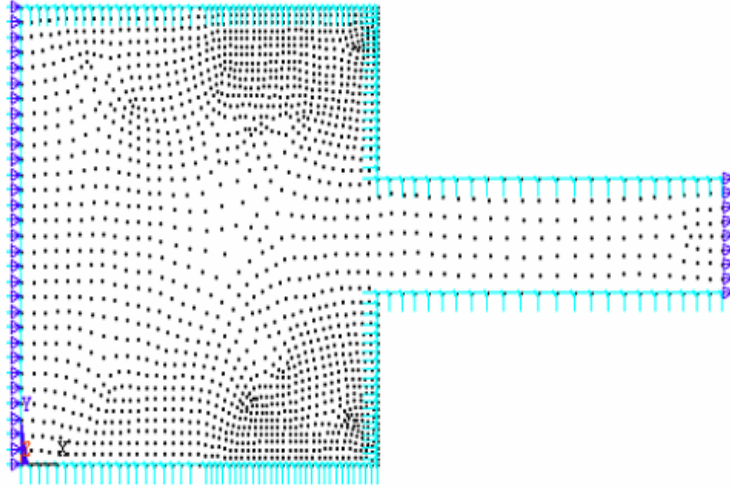
Firstly, a three-dimension (3D) geometry model of the entry flow in a channel is constructed in ANSYS window, and then the type of element is defined as 3D FLOTRAN141, as shown in Figure 2. The geometry model is divided a number of meshes. Line dividing density is set up manually. The mesh type is square, and is divided freely. The results are also shown in Figure 2.



**Figure 2.** 3D FEM model and meshing.

### 3.2. Loading

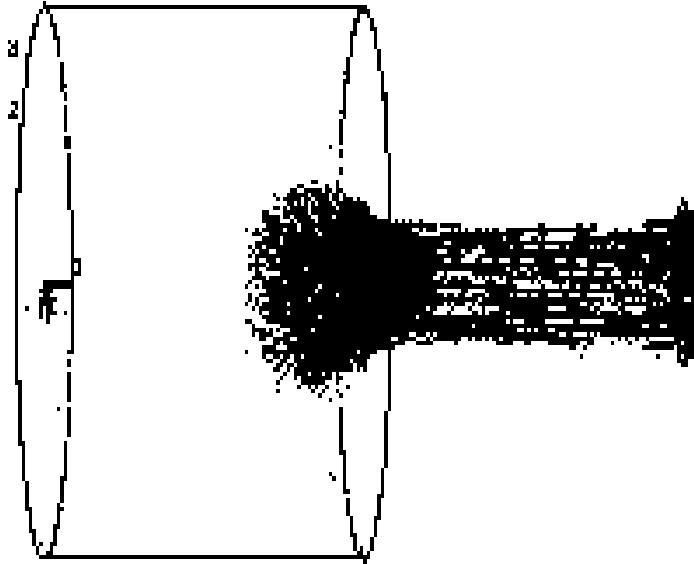
During this finite element simulation, the initial conditions are exerted to the geometry model. The pressure at the entrance is 20MPa, the temperature of the materials is 90°C, the velocities at radial and axial direction in boundary are set up as zero, the initial velocity at the entrance is 2mm/s, 5mm/s, and 10mm/s, respectively, and the corresponding fluid viscosities are 15, 10, and 7kPa·s, respectively. The detail is shown in Figure 3.



**Figure 3.** Loading of model.

### 3.3. Solution

Flow velocity distribution chart displays the velocity change in the channel vividly. Using the solution software to solve this flow question, and then we can enter post-treatment stage, draw the charts of flow velocity distribution, as shown in Figure 4. It can be seen that there is obvious entry converging flow phenomenon.



**Figure 4.** Flow velocity distribution in inlet region.

#### 4. Results and Discussion

##### 4.1. Influence of initial velocity on flow field

The discussion of the influence of initial velocity ( $V_o$ ) on the velocity field is based on the coordinate systems as shown in Figure 5. Figure 6 illustrates the effect of initial velocity on the velocity distribution at the radius direction in the front region of the entrance. It can be seen that the velocity change rate increases with increasing  $V_o$  from the wall to the center of channel, velocity at the wall is zero, and it reaches the maximum at the channel center. In addition, the velocity distribution of channel section changes with varying  $V_o$ . With the increase of  $V_o$ , the curve curvature of the velocity distribution increases step by step. Namely, the shape of the curve changes from flatness to urgently step. In the obvious work, the author [8] derived an entrance converging flow boundary streamline equation according to the minimum energy theory, and proposed an extensional strain rate expression as follows:

$$\frac{dV_z}{dz} = \frac{1}{2} \left( \frac{2\eta_e}{\eta_s} \right)^{\frac{1}{2}} \dot{\gamma}_w \left( \frac{R}{r} \right)^{\frac{3n+1}{2}}, \quad (1)$$

where  $n$  is the non-Newtonian index (an important parameter for characterization of fluid non-Newtonian property),  $\dot{\gamma}_w$  is the shear rate,  $R$  is the radius of channel,  $\eta_e$  and  $\eta_s$  are the extensional viscosity and shear viscosity, respectively.



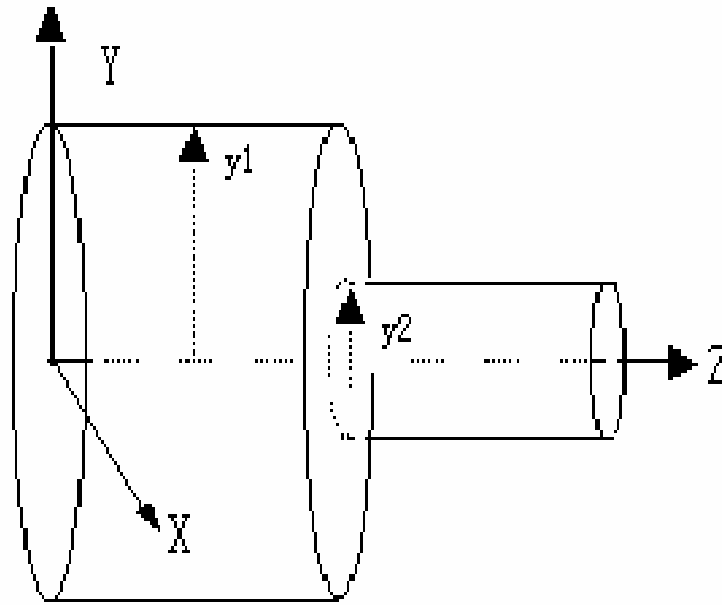


Figure 5. Sketch of 3D coordinate.

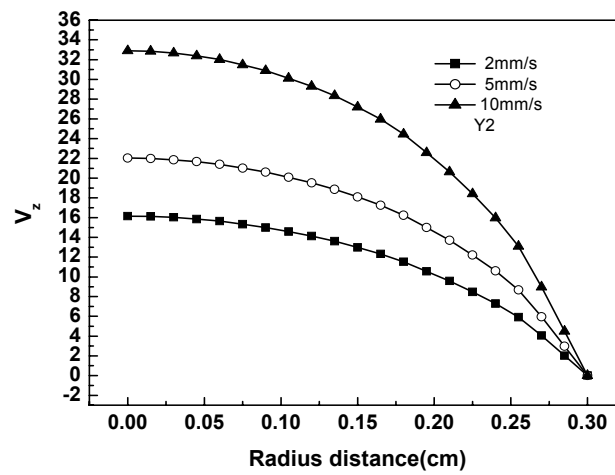
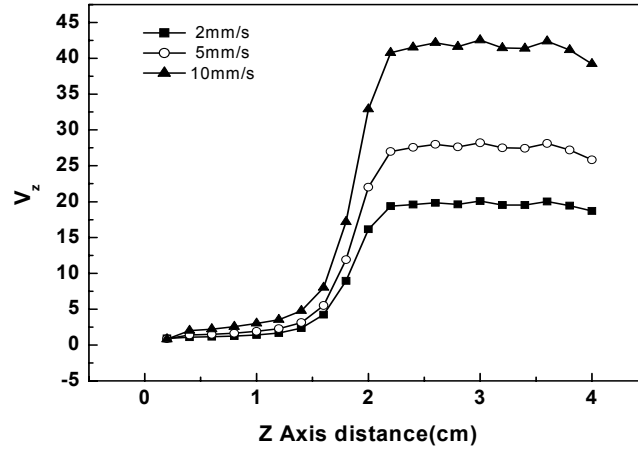


Figure 6. Flow velocity distribution curves at radius direction.

It is known from Equation (1) that the velocity increases non-linearly with decreasing the channel radius, and it reaches the maximum when the radius tends to zero (i.e., tending to the channel center), this is similar to the simulation as shown in Figure 6. That is, the velocity is a non-linear function of the radius when the other parameters are constant, which is given by

$$V_z = f\left\{\left(\frac{1}{r}\right)^{\frac{3n+1}{2}}\right\}. \quad (2)$$

Figure 7 demonstrates the influence of the initial velocity on the axes flow velocity distribution. With increasing the axial flowing distance, the axes flow velocity increases non-linearly. When the location is close to the entrance, the flow velocity increases suddenly, and then the flow velocity varies slightly. In addition, with the increase of  $V_o$ , the increase of the flow velocity raises correspondingly.

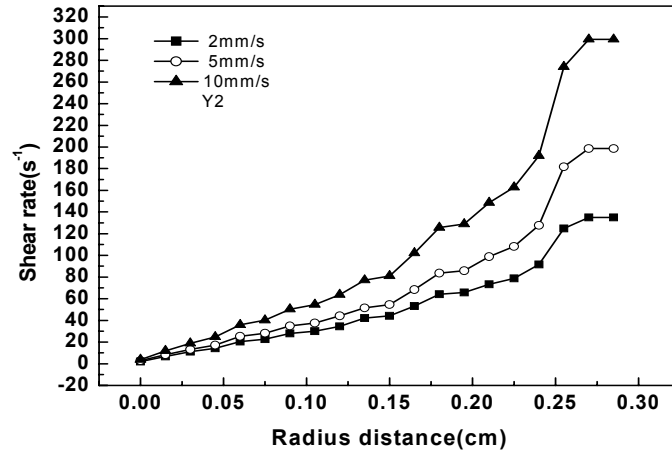


**Figure 7.** Flow velocity distribution curves at axes direction.

Equation (1) describes the relationship between the extensional strain rate and the rheological property parameters and shear rate as well as the channel geometry parameters. It may be observed from Figures 6 and 7 that the velocity distribution described by means of Equation (1) is similar to the simulation.

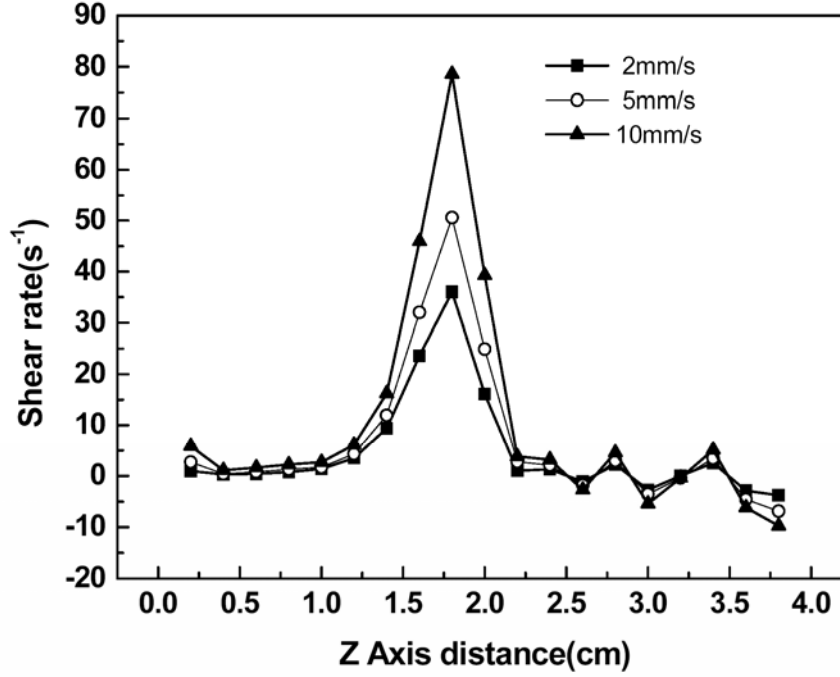
#### 4.2. Influence of initial velocity on shear rate

Figure 8 illustrates the influence of the initial velocity on the shear rate at radius direction during the entry flow of the fluid. It can be seen that the shear rate increases non-linearly with increasing the radius distance from the center axes of the flow channel. Similarly, the shear rate increases with an addition of the initial velocity, especially in the case of the radius distance being large.



**Figure 8.** Shear rate distribution at radius direction.

Figure 9 displays the influence of the initial velocity on the shear rate at the channel axial direction. It can be seen that the shear rate in the front district of the inlet increases non-linearly with an increase of the axial distance along  $z$  direction, and it reaches peak value at the entrance. This is because that entrance convergence induces the velocity of the melt increase sharply here, resulting in suddenly changing the velocity rate. For a given viscoelastic fluid, the entry converging flow enhances with increasing flow velocity when extrusion conditions are fixed, leading to expanding the circulation district correspondingly [8, 10]. Consequently, the extensional strain rate increases relevantly.



**Figure 9.** Shear rate distribution at axial direction.

Liang [11], [14] studied the effects of extrusion conditions on planar entry converging flow patterns of rubber compound. On the basis of a boundary streamline differential equation published before, he proposed a formula for estimating the length ( $L_c$ ) of circulation district as follows:

$$\frac{L_c}{2R} = \frac{0.5e\beta[(1/\beta)^{(3n-1)/2} - 1]}{\frac{2(3n-1)}{3(n+1)} \left( 1 - \beta^{3(n+1)/2} + \frac{(3n-1)\xi}{3(n-1)} \left[ (1/\beta)^{3(n-1)/2} - 1 \right] \right)}, \quad (3)$$

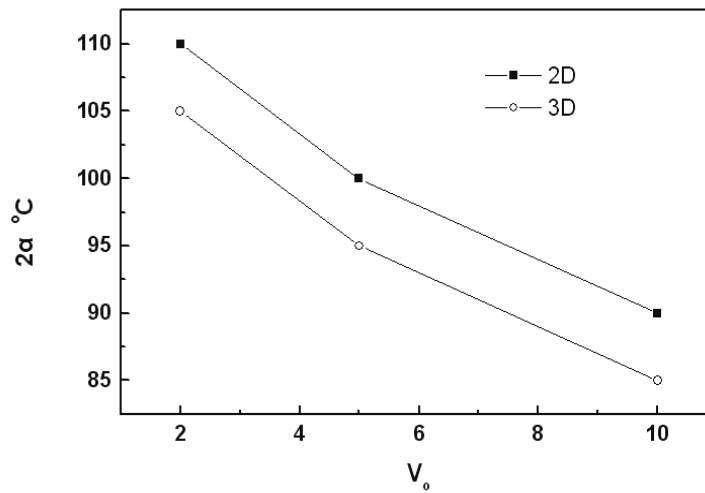
where  $e$  is the Bagley correction factor (an important parameter for characterization of the entry elastic storage energy of fluid),  $\xi$  is the coefficient related with the fluid adhesion characteristic,  $\beta$  is the reciprocal of die contraction ratio, and  $n$  is the non-Newtonian index.

From above equation, we can see that entry converging flow patterns (such as the size of vortex zone) related closely the entry elastic deformation energy and non-Newtonian index under given conditions.

### 4.3. Influence of initial velocity on convergence angle

The dependence of the natural entry converging half angle ( $\alpha_0$ ) of viscoelastic fluids on the initial velocity is showed in Figure 10. It can be seen that the natural entry convergence half angle reduces with increase of the initial velocity. It is because that the increase of the initial velocity makes velocity field drawn along the axial, the natural entry convergence angle reduce correspondingly. With the increase of initial velocity, the reducing range will diminish gradually. The author [8] used the minimum energy theory in the variation calculus, derived a mathematical model, which predicts  $\alpha_0$ :

$$\alpha_0 = \operatorname{tg}^{-1} \left\{ \frac{2}{3e} \left[ \frac{2}{n+1} + \frac{\xi}{1-n} \right] \right\}. \quad (4)$$



**Figure 10.** Dependence of entrance natural convergence angle on initial velocity.

When temperature is constant, the entrance convergence flow of the fluid will be enhanced with an increase of the flow velocity, the fluid elasticity stored energy may increase thereupon in the entry, presenting as the increase of the  $e$  value within the specific limits. Meanwhile, the viscous dissipation of deformation energy during entry flow increases to some

extent ( $n$  value increases). Therefore, the term  $2/(n+1)$  value in formula (3) decreases with increase of the  $n$  value, whereas the value of  $\xi/(1-n)$  increases at the same time, so the influence of change of  $n$  on  $2\alpha_0$  value is relatively small. In other words,  $2\alpha_0$  depends on  $e$  mainly. We can also see from Figure 10 that, the  $2\alpha_0$  value of the three-dimension (3D) is lower than two-dimension (2D) under the same initial velocity.

Simulation results show that the vortex zone area increases with the increase of the flow velocity. Under the actual process conditions, the  $2\alpha_0$  value of rubber/carbon black compounds is about  $45\sim 75^\circ$ . It is, therefore, known from Figure 10 that, the simulation results by three-dimensional finite element is closer to reality than those by two-dimensional simulation.

#### 4.4. Discussion

From Equation (1), the initial velocity is the function of the rheological properties (e.g., extensional viscosity and shear viscosity) and the axial distance under given conditions. Moreover, the extensional viscosity and shear viscosity is the function of the axial distance. Namely, the velocity is the functionals of the axial distance, that is,

$$V_z = f\{\eta_e(z), \eta_s(z), z\}. \quad (5)$$

This indicates that the relationship between the  $V_z$  and  $z$  is quite complicated. When the melt is near the channel inlet, the converging flow is enhanced, leading to the  $V_z$  increases quickly (see Figure 7).

Equation (2) describes the relationship among the ratio of the length of circulation district to the channel radius ( $\frac{L_c}{2R}$ ) and the rheological properties of polymer melts including the Bagley correction factor and the fluid adhesion coefficient, as well as the channel geometry parameters such as the channel contraction ratio. White and Baird [21] observed the planar entry flow behavior of LDPE and polystyrene (PS) melts by means

of flow visualization, and found that the  $\frac{L_c}{2R}$  increased non-linearly with increasing shear rates, and the values were from 0.1 to 0.45 in a range of shear rate from 1 to 100s<sup>-1</sup>. This is close to the predictions with Equation (3) [11].

According to the definition of apparent shear rate of the fluid, the apparent shear rate in entrance flow may be expressed as follows:

$$\dot{\gamma} = \frac{4V_z r^2}{R^3}, \quad (6)$$

whereas  $r$  is the radius in the converging flow region, namely, the distance from the center to the converging boundary streamline at a certain position. Hence, it is the function of the axial distance under the given conditions such as initial velocity and channel contraction ratio, that is,

$$\dot{\gamma} = f\{r(z)\}. \quad (7)$$

It is known from Equations (6) and (7) that the shear rate tends to zero when the  $r$  is near the channel center. This is close to the simulations (see Figure 8). The  $V_z$  reaches the maximum at the channel entrance, resulting in the shear rate generating a peak in this case (see Figure 9).

Equation (4) describes the relationship between the half entry natural converging angle and the rheological properties of polymer melts including the Bagley correction factor and the fluid adhesion coefficient, as well as the non-Newtonian index. In the previous work, the author [13] estimated the entry natural converging angles during capillary extrusion flow of carbon black filled NR/SBR compound using Equation (4), the results showed that the  $\alpha_0$  decreased with increasing shear rate, and the value was about 68~45° when the shear rates were from 90 to 230s<sup>-1</sup>. These are close to the simulation results shown in Figure 10.

### 5. Conclusion

The simulated axes flow velocity increased non-linearly with increasing the initial velocities ( $V_o$ ) and reached the maximum at the center and the inlet of the channel. The simulated shear rate increased non-linearly with increasing the  $V_o$  and the radius distance from the center axes to the wall of the flow channel, and the shear rate reached the maximum at the entrance. The simulated half natural convergence angle  $\alpha_0$  reduced non-linearly with an increase of the  $V_o$ , and the  $2\alpha_0$  value of the three-dimension (3D) simulation was lower than two-dimension (2D) simulation under the same initial velocity.

The velocity distributing, the variation of extension rate and shear rate as well as  $\alpha_0$  simulated by means of a finite element method during the entrance converging flow of polymer fluids were close to the experimental observations and the estimations by using Equations (1)~(7) proposed in literature.

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